| Δ | 400 | liter tank initially | holds 40 | liters of brine | containing 1 | gram of salt per liter. |
|---|-----|----------------------|----------|-----------------|--------------|-------------------------|
| H | 400 | mer tank initially   | noids 40 | mers of brine   | containing 1 | gram of sait per mer.   |

SCORE: \_\_\_\_ / 12 PTS

Brine containing 2 grams of salt per liter starts flowing into the tank at 6 liters per minute.

At the same time, the well-mixed solution leaves the tank at 4 liters per minute.

[a] Find the amount of salt in the tank t minutes after the brine starts flowing into the tank, assuming the tank does not overflow. HINT: Simplify all fractions as soon as possible.

[b] BONUS: (the case in which the rate at which brine enters, and solution leaves, are not constant)

= 4(20+t) + C(20+t)

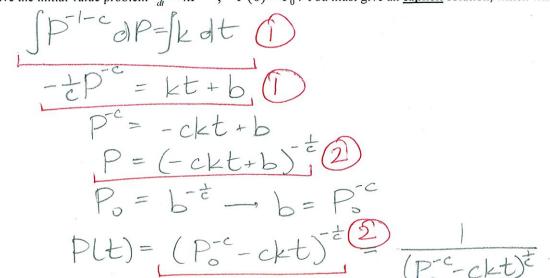
SCORE: \_\_\_\_\_ / 3 PTS

If brine enters the tank at I(t) liters per minute, and solution leaves the tank at O(t) liters per minute, and all other information remains the same as above, write <u>BUT DO NOT SOLVE</u> an initial value problem for the amount of salt in the tank t minutes after the brine starts flowing into the tank, assuming the tank does not overflow.

$$\frac{dx}{dt} = 2I(t) - O(t) \frac{x}{40 + \int_{0}^{t} I(u) - O(u) du} (2)$$

The population model  $\frac{dP}{dt} = kP^{1+c}$  (where k, c > 0) is called a doomsday equation, because solutions of the equation go to infinity in a finite time. (In contrast, the exponential model in class goes to infinity in infinite time.)

[a] Solve the initial value problem  $\frac{dP}{dt} = kP^{1+c}$ ,  $P(0) = P_0$ . You must give an <u>explicit</u> solution, which will involve k and c.



[b] Find the time at which doomsday occurs (the population goes to infinity).

The population model  $\frac{dP}{dt} = (a - b \ln P)P$  is known as the Gompertz differential equation (where a, b > 0). SCORE: \_\_\_\_\_/9 PTS Solve that differential equation with the initial condition  $P(0) = P_0$ . You must give an <u>explicit</u> solution, which will involve a and b.

$$P(a-b|nP) dP = \int dt (2)$$

$$L u = a-b|nP$$

$$-b|n|a-b|nP| = t+c (3)$$

$$a-b|nP = Ce^{-bt}$$

$$a+Ce^{bt} = b|nP$$

$$|nP = \frac{a}{b} + Ce^{-bt}$$

$$P = e^{\frac{a}{b} + Ce^{-bt}} (2)$$

$$P = e^{\frac{a}{b} + C}$$

$$P = e^{\frac{a}{b} + Ce^{-bt}} (2)$$

$$P = e^{\frac{a}{b} + Ce^{-bt}} (2)$$